Super Matrix Solver-AMG

Fast & Robust Sparse Matrix Solver

Product Introduction

April, 2011

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VPD-F11002=issue3(L2)

VINAS
Visual Integration & Numerical Analysis Systems

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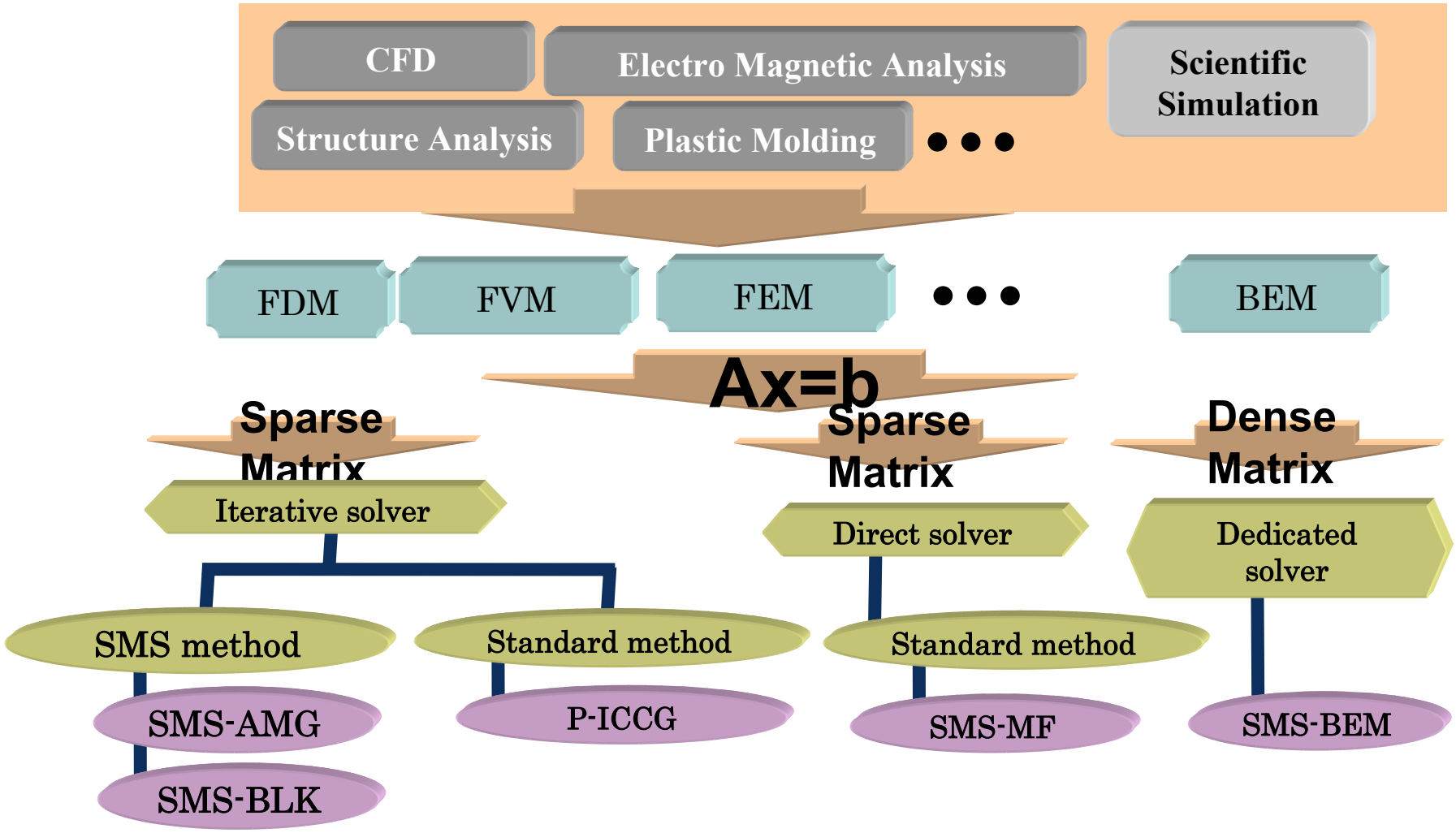
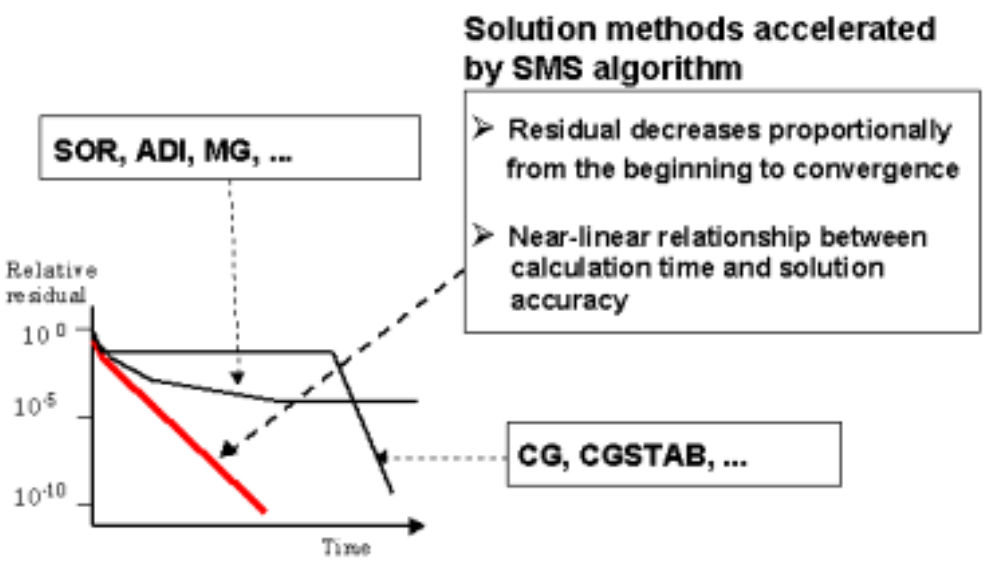
Super Matrix Solver Library



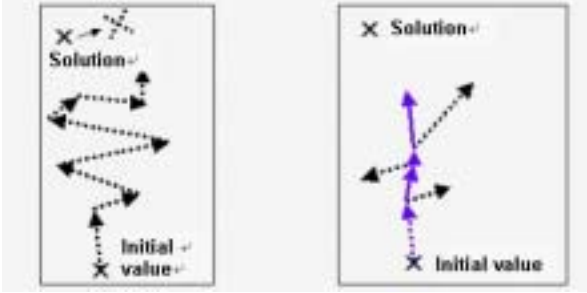
What is *Super Matrix Solver* (*SMS*)?

- ✓ Jointly developed with National Aerospace Laboratory of Japan
- ✓ Acceleration technology for iterative matrix solution methods.
- ✓ It can be incorporated into various existing solution methods to accelerate and stabilize process of matrix calculation.



Algorithm of SMS

■ Super Matrix Solver has an optimization algorithm to accelerate and stabilize solution process of iterative methods such as SOR and AMG.



Conventional iterative methods | Super Matrix Solver

What is *Super Matrix Solver-AMG*?

General AMG method

- ✓ AMG method stands for Algebraic Multi Grid method, which is an iterative method developed during 1980-1990 based on the latest calculation theories.
- ✓ It is a fast calculation method, but not widely used because of following problems.
  - ✓ **Difficult to program. Needs advanced mathematical knowledge.**
  - ✓ **Difficult to find out best combination of parameters due to large number of parameters to set.**

Super Matrix Solver-AMG

- ✓ Matrix solver based on AMG method and speeded-up and stabilized by Super Matrix Solver technology of VINAS.
- ✓ One module of Super Matrix Solver library.

Benefits of using *Super Matrix Solver*

- ✓ Speed-up the calculation process by numerical analysis programs without enhancing computer hardware resources
- ✓ Obtain converged solution for problems that have never been solved by conventional methods due to divergence and robustness of high-speed calculation methods
- ✓ Obtain accurate solutions for very large models in a stable way
- ✓ Minimize manual setting of parameters

Issues in matrix calculation

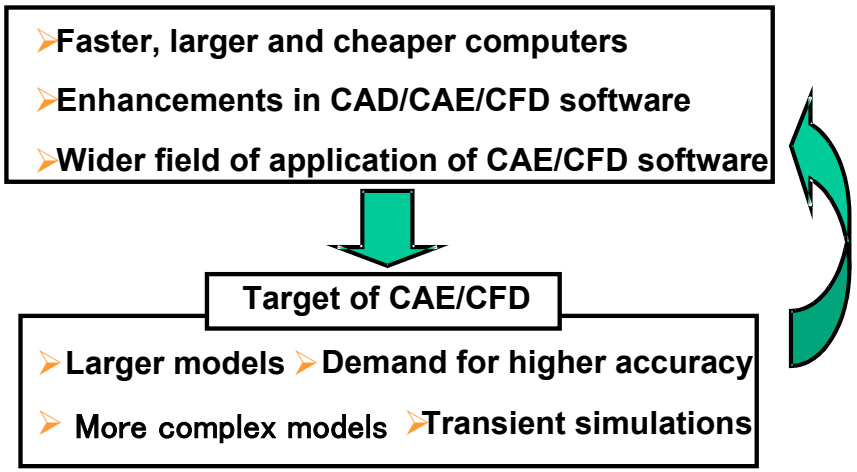
✓ Enhance Calculation Speed

Contributes directly to speeding-up the whole CFD solver process

- Increase convergence rate
- Implement parallel processing

✓ Improve robustness and convergence rate

- Prevent divergence of calculation
- Prevent stagnation of convergence



✓ Usability enhancements

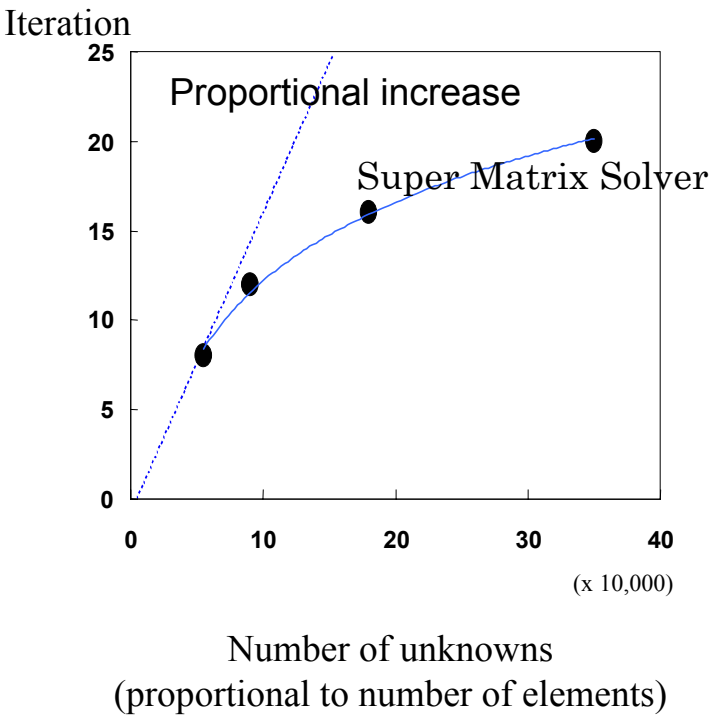
- Reduce number of parameters to set
- Introduce easier way of convergence judgement

✓ Improve accuracy of Numerical Analysis

Essential in large calculations

- Eliminate cancellation of significant digits
- Reduce accumulation of errors

Special Feature of *Super Matrix Solver (SMS)*



1) Fast calculation

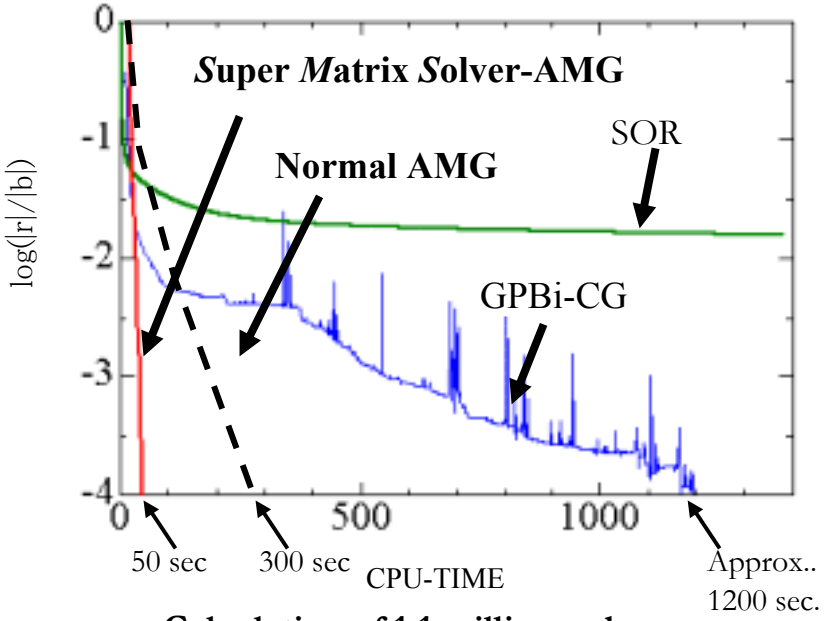
2) High stability and
high convergence rate

3) Reduced parameter setting

4) Not liable to error accumulation
- Keeps accurate solution
- Suitable to very large calculations

5) Comparatively less iterations
needed for large calculation

Performance of *Super Matrix Solver-AMG*



Calculation of 1.1 million unknowns

Logarithm of relative residual vs. CPU time for calculation

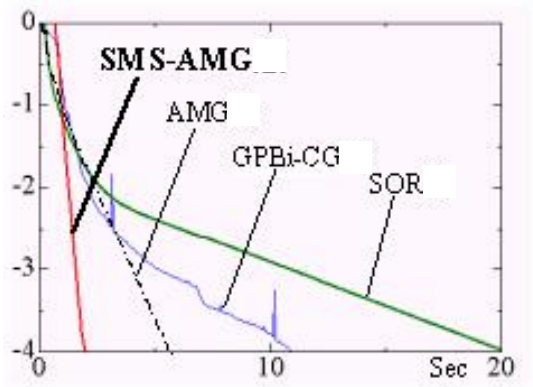
＊ GPBi-CG is relatively fast and robust among CG methods

***Super Matrix Solver-AMG* and other solver methods**

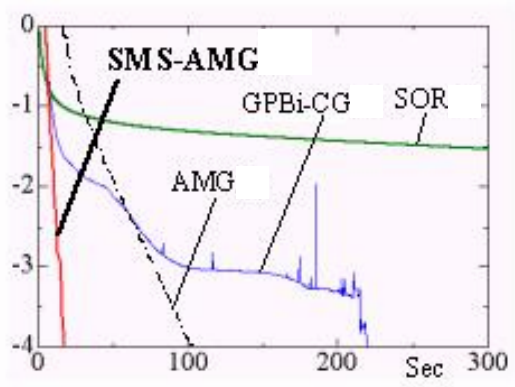
Advantages of *Super Matrix Solver*-AMG

- Based on AMG method and is made faster and more robust
- Much faster than CG methods
- Problems that have never been solved by other methods can be robustly solved
- Very stable convergence during calculation

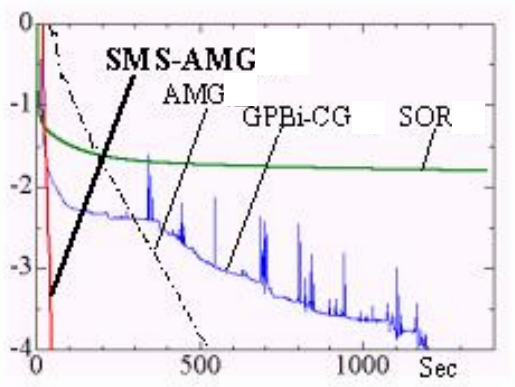
Performance of *Super Matrix Solver-AMG*



CASE1 :0.05 Million Elements



CASE2 :0.3 Million Elements

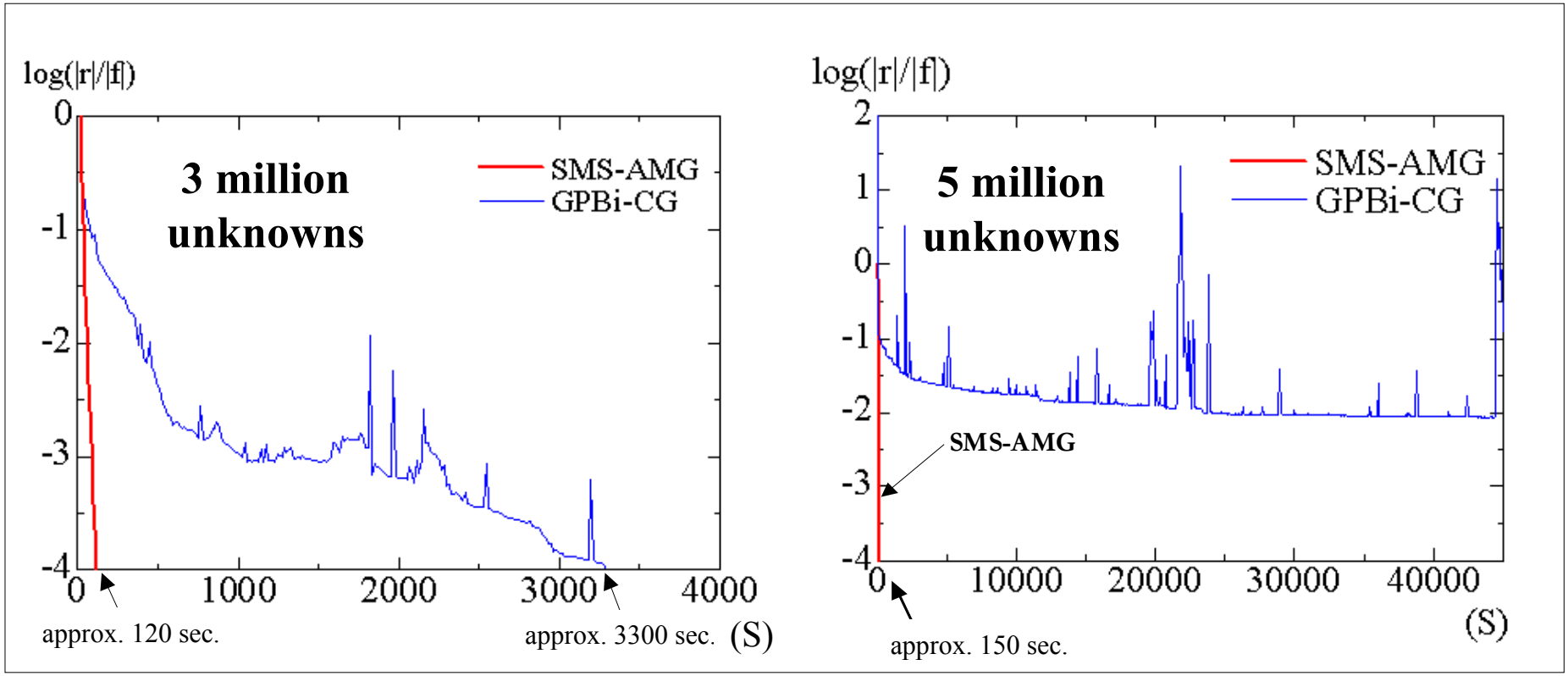


CASE3 :1.1 Million Elements

Results of convergence history of simultaneous linear equation by Super Matrix Solver-AMG, normal AMG, normal SOR, and GPBi-CG solvers are compared above. Reduction of common logarithms of relative residuals (ordinate) versus CPU time (abscissa) is plotted. Calculations were stopped when relative residuals reached 1.0 x 10e-4.
* GPBi-CG is a solution method known by its speed and stability among CG methods.

Performance of *Super Matrix Solver-AMG*

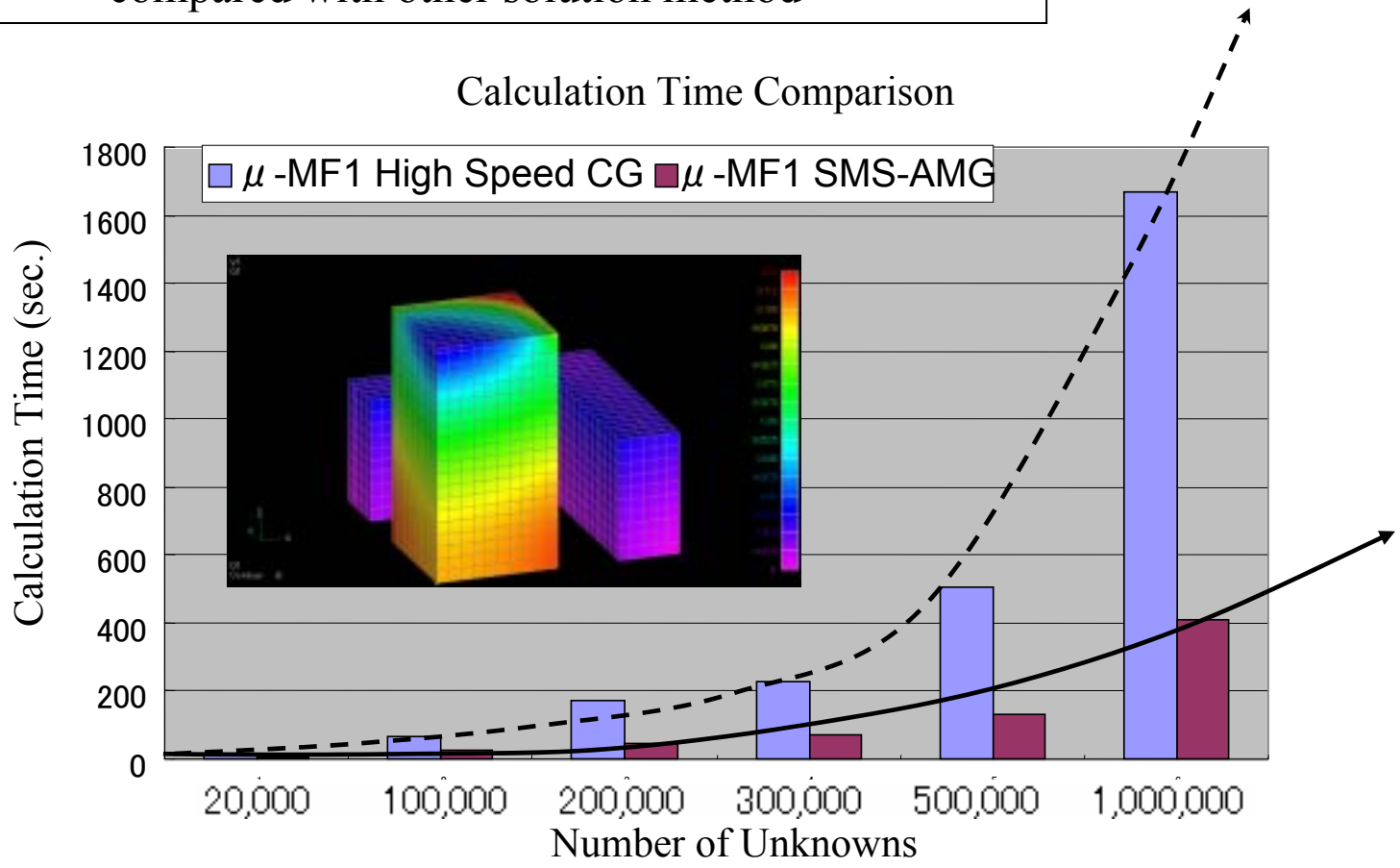
Characteristic : Calculation time does not increase proportionally to problem size increase



Only 30 seconds of calculation time increase for an increase of 2 million in the number of unknowns

Performance of *Super Matrix Solver-AMG*

Characteristic : Less calculation time increase for larger problems compared with other solution method

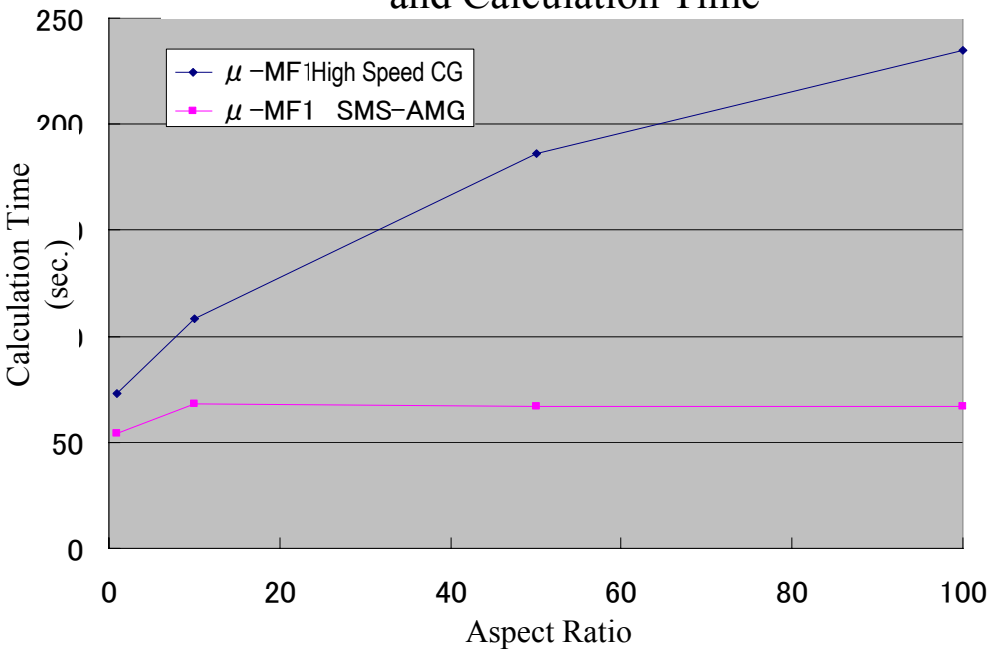


Example of Electromagnetic Analysis by MU TECH Co., Ltd.

Performance of *Super Matrix Solver-AMG*

Stably calculates lower quality mesh

Relations betw. Aspect Ratio of Finite Element and Calculation Time



	Number of Unknowns	Aspect Ratio	Gird Condition
CASE1	200000	1:1	coarse
CASE2	200000	1:10	sort of coarse
CASE3	200000	1:50	right coarseness
CASE4	200000	1:100	too fine

	Calculation Time (sec.)		Iteration Count (times)		Calculation Precision (G)	
	CG	SMS	CG	SMS	CG	SMS
CASE1	73	54	388	21	211.7	211.7
CASE2	108	68	692	24	196.0	196.0
CASE3	186	67	1323	26	200.0	200.0
CASE4	235	67	1747	25	200.6	200.6

As gird condition worsens, so does the calculation difficulty...

Confirmed that iteration count of SMS-AMG does not depend on aspect ratio

Example of Electromagnetic Analysis by MU TECH Co., Ltd.

Specifications of *S*uper *M*atrix *S*olver-AMG (1)

- Type of matrix: Sparse matrix
  Supports calculation of structured and unstructured grid in CFD and other analyses
  Supports calculation of asymmetric matrix
- Zero diagonal elements: Can not calculate
- Number of unknowns: No limitation
- Type of variables: (single) and double precision versions

Specifications may change without prior notice.

Specifications of *S*uper *M*atrix *S*olver-AMG (2)

Input parameters (*1)	Left side coefficient matrix (A), Right side constant vector (b) , convergence criteria, maximum iterations, ...
Output data	Solution vector(X), Relative residual, Number of performed iterations, ...
Error messages	Warnings and error messages are returned as return values (calculation information, system information, ...).

Specifications may change without prior notice.

Specifications of *S*uper *M*atrix *S*olver-AMG (3)

Supported systems	Windows, Unix(SGI/IRIX, ...) Linux and parallel versions planned * Consult us for details
Provided as	Library (source is not open)
Documents	Manual (explains data format, parameters, application development, and so on)
Sample data	Sample program using ***S*uper *M*atrix *S*olver-AMG**
Field of application	Can be used for solving linear equation system in CFD, structural, electromagnetic and other analyses.
License management	Managed by physical address of computer Runs on designated machine only

Specifications may change without prior notice.

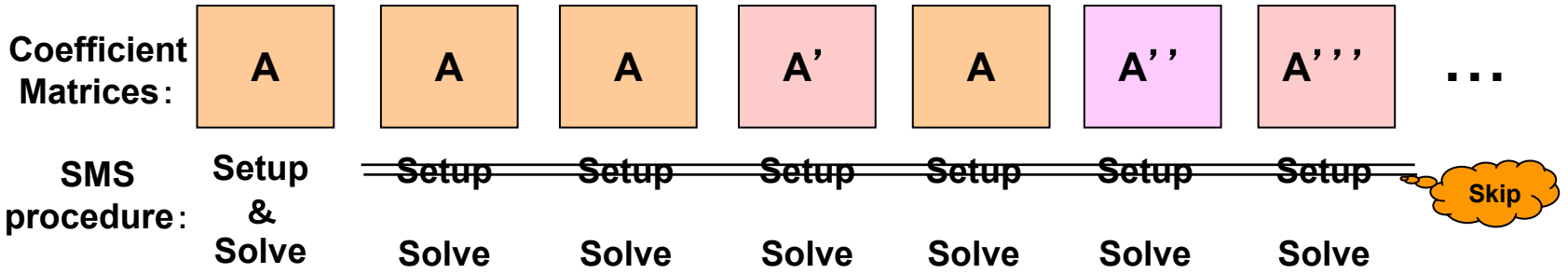
Updated feature of *SMS-AMG Version 3*(1)

Up to 5 sets of setup information can be saved

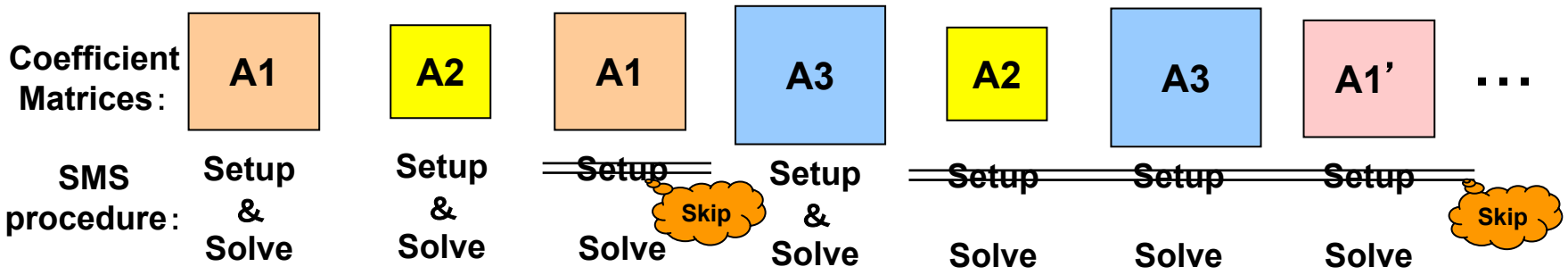
Speed-Up

- Computational time can be significantly reduced by skipping setup process (creating coarse grids, etc.) when solving the same or similar* coefficient matrix for different right hand side vectors

  (* the position of nonzero elements are the same but the values are different)



- Multiple sets of setup information can be saved; applications in which multiple coefficient matrices need to be solved are supported

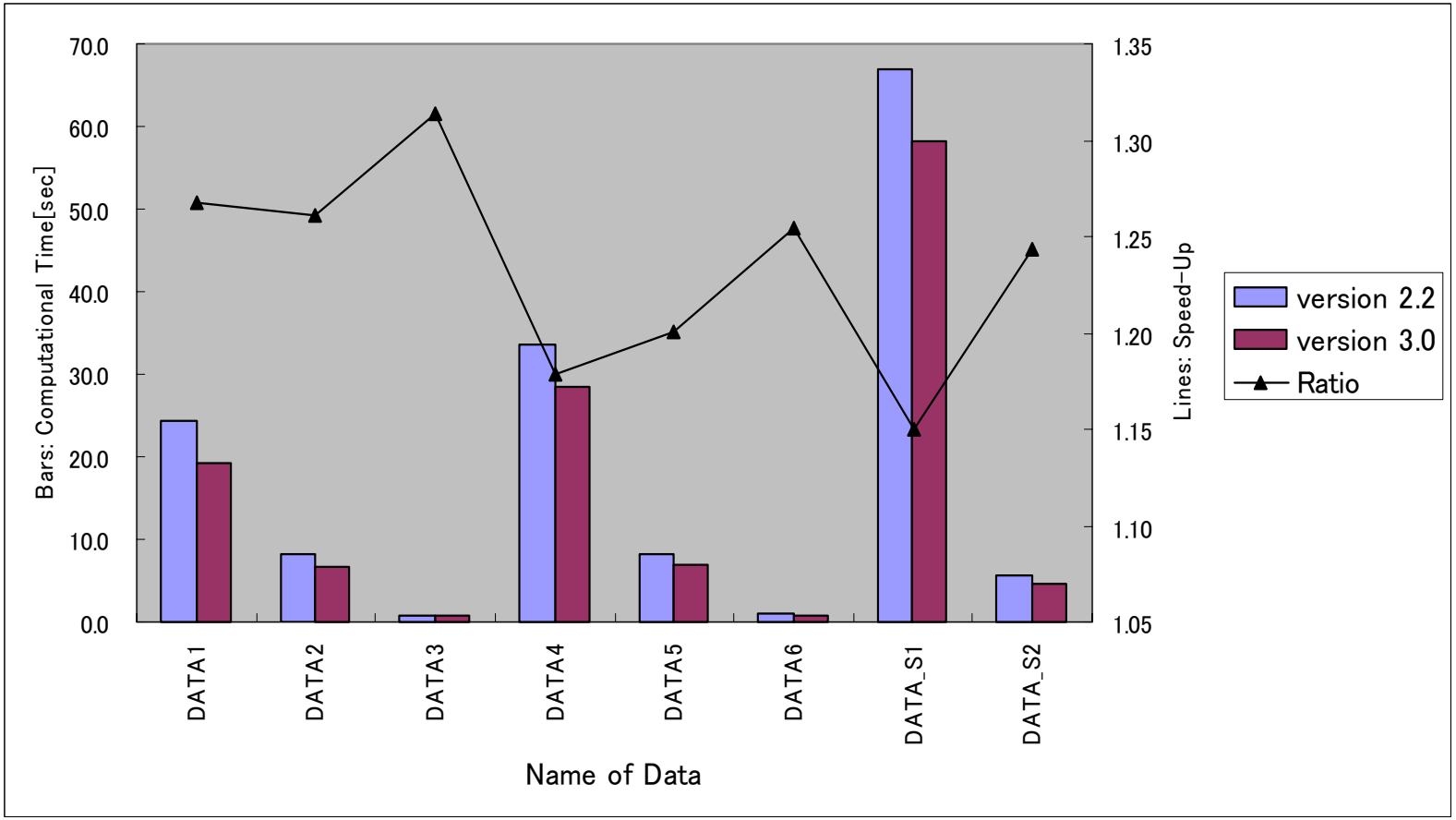


Updated feature of *SMS-AMG Version 3*(2)

Speed-Up

Speed-Up

20～30% Speed-Up compared to Version 2.2 (on Linux 32bit)



Performance of Version 3 (compared to Version 2.2 released on Sep. 2005)

Updated feature of *SMS-AMG Version 3*(3)

Memory saving

Improved memory efficiency for symmetric matrices

Only upper-half is stored; memory consumption is reduced by 30~40%

example: memory consumption (of the whole application(*1))

	Whole Matrix	Upper half	Memory reduction rate
DATA4 (186843dofs)	682.0MB	384.9MB	43.6%
DATA5 (56979dofs)	214.9MB	124.9MB	41.8%
DATA6 (12288dofs)	51.5MB	35.2MB	31.6%

(*1)The sample program attached to the SMS product, which is an application that just solves a matrix equation

Robustness

Improved robustness by introducing choice between CG and RC methods when solving symmetric systems

Some Problems that cannot be solved with Version 2 can be solved with Version 3

■National Laboratories, Universities

Central Research Institute of Electric Power Industry	CFD
Fukui University	CFD
Hokkaido University	Structural Strength (Breakage)
Japan Aerospace Exploration Agency	CFD
Japan Atomic Energy Institute	CFD
Japan Nuclear Cycle Development Institute	CFD
Kyusyu Institute of Technology	Electromagnetic Analysis
Kyusyu University	CFD
National Institute of Materials Science	Structural Analysis (Molecular Level)
National Maritime Reseatch Institute	CFD
Okayama University	CFD
Osaka University	Structural Analysis
Port and Airport Research Institute	CFD
Saitama Institute of Technology	CFD
Shizuoka University	CFD
The Institute of Physical and Chemical Research	CFD
The University of Tokyo	CFD
Tokyo Institute of Technology	CFD
Yokohama National University, etc	CFD

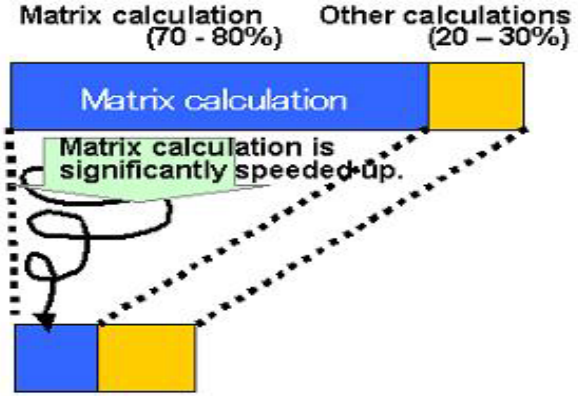
■Industry

CANON INC.	CFD
CRC Solutions Corp.	CFD
Denso Corporation	Plastic Mold Flow Analysis
Fuji Research Institute Corp.	CFD
Hitachi, Ltd.	CFD
Kajima Corporation	Structural Analysis
Kobe Steel, Ltd.	Plastic Mold Flow Analysis
Matsushita Electric Industrial Co., Ltd.	Electromagnetic Analysis
Mazda Motor Corporation	Aerodynamics Analysis
Nikon Corporation	Electromagnetic Analysis
Panasonic	Plastic Mold Flow Analysis
Plamedia Corporation	Plastic Mold Flow Analysis
Ricoh Co., Ltd.	Electromagnetic Analysis
Shimizu Corporation	Groundwater Flow Analysis
Sumitomo Metal Industries, Ltd.	CFD
Toray Industries, Inc.	Plastic Mold Flow Analysis
Toshiba Tec Corporation	CFD (Two-phase flow)
μ-Tech Inc., etc	Electromagnetic Analysis

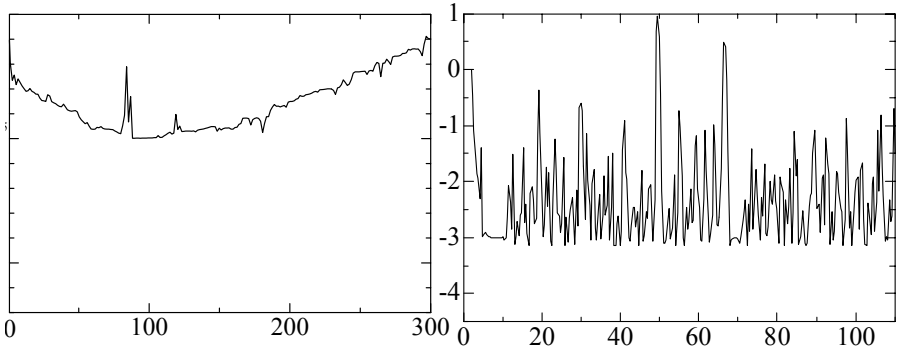
Performance and Application Examples

Benefits of *Super Matrix Solver*

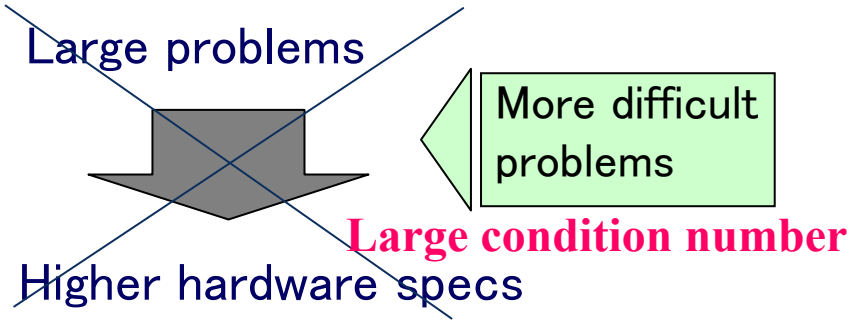
Faster numerical analysis calculations



Solutions to previously insoluble problems



Highly accurate solutions can be obtained for very large scale problems



Minimized manual settings of calculation parameters

rtc=smsamgd(solution, coefficient matrix, right-hand side vector, number of iterations, normalization, convergence criterion, reduction)

National Maritime Research Institute Maritime Safety Department

Numerical analysis model: LES-Smagorinsky model

Numerical method:

- Non-uniform meshes in the Cartesian coordinate system
- Discretization on staggered grid
- Calculates time-expansively by Fractional Step method
- Time integration: Second-order Adams-Bashforth scheme
- Discretization of space differentiation term: Second-order central difference (Uses difference scheme Suitable for Non−uniform grid)

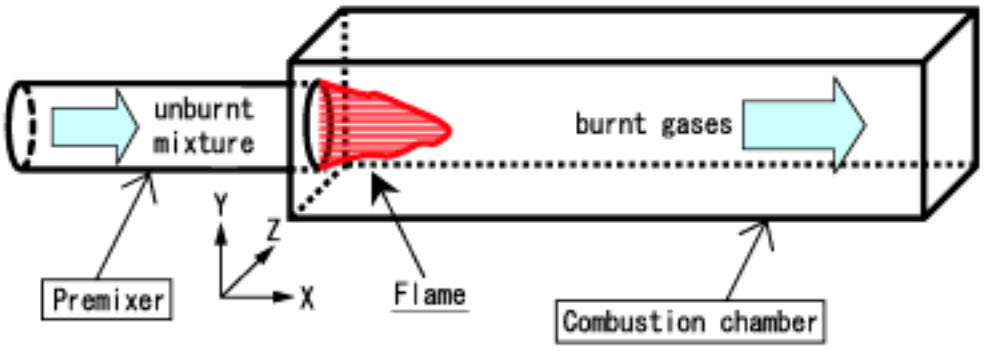


Table 1: Conditions of simulation

Reference velocity (U)	11.0 [m/s]
Reference length (L)	5.0 [cm]
Reference acoustic pressure (p^0)	0.1 [MPa]
Laminar burning velocity (S_L)	0.33 [m/s]
Fresh gases temperature (T_u)	300 [K]
Burnt gases temperature (T_b)	1430 [K]
H_2/air equivalence ratio (ϕ)	0.4
Computational domain (X×Y×Z)	10.0×2.0×2.0
Number of grid points	96×48×48
Reynolds number ($Re = \rho_u UL/\mu_u$)	34300
Prandtl number ($Pr = \mu_u/\rho_u\lambda_u$)	0.7
SGS Prandtl number (Pr_t)	0.9
SGS Schmidt number (Sc_t)	0.9

National Maritime Research Institute Maritime Safety Department

Work by VINAS

1）Investigated the portion regarding matrix calculation of the original program code.

2）Changed the original program in order to check the performance (calculation time by BiCGSTAB) before integrating SMS-AMG, and did the check.

3）**Changed the original program in order to integrate SMS-AMG.**

Calculates without making coefficient matrix A and right hand side vector b.

Makes coefficient matrix A and right hand side vector b, and then calculates passing them to SMS-AMG.

4）Specially tuned SMS-AMG up for more efficient calculation.

Pressure Poisson equation

$$\nabla^2 \bar{p}^{n+1} = \frac{1}{\Delta t}\left[\nabla\left(\bar{\rho}\tilde{u}\right)^* + \left(\frac{\partial \bar{\rho}}{\partial t}\right)^{n+1}\right]$$

Original code

In-line type:
BiCGSTAB

Calculates pressure p at each grid point without making A and b.

Changed code

Subroutine type:
SMS-AMG

Makes A and b, and then calculates pressure p of all grid points at once calling SMS-AMG.

National Maritime Research Institute Maritime Safety Department

Improvement by integrating SMS-AMG

Time comparison of turbulent combustion analysis after expanding enough.
(time interval: 5 ms, 1000 iterations)

OS: Linux (Pentium III, clock=1.13GHz, memory=1GB)

	Time[min.]		Improvement by SMS-AMG	
	BiCGSTAB	SMS-AMG	Time elimination rate [%]	Processing speed ratio
Whole process	426.3	320.0	24.9	1.33
Matrix calculation	144.2	37.1	74.3	3.89

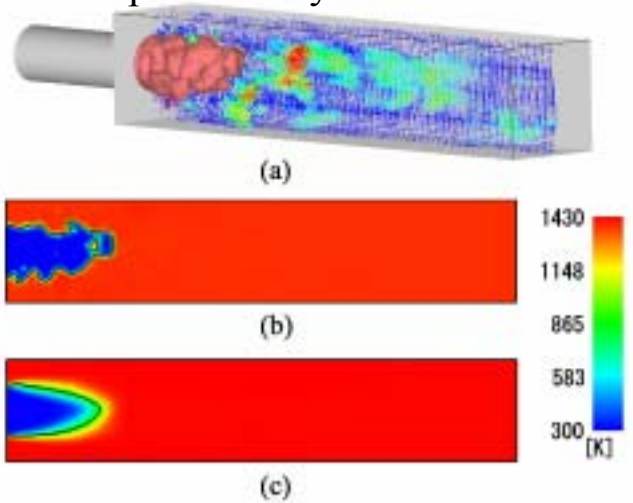
Customer's voice

Matrix calculation speed was improved about four times faster than the original program (about twenty times faster than a program using the SOR method) although elimination ratio of whole processing time was small because this analysis contains a special calculation process. This improvement boosted up efficiency of our study. By the elimination of calculation time, we became to be able to increase number of grid and analyze turbulent combustion in more detail.

References:

「Large Eddy Simulation of Premixed Turbulent Combustion Using the Flamelet Model Based on the G-equation」 - Hideyuki OKA
Theoretical and Applied Mechanics Japan, Vol.53
October 2004

Example of analysis result



(a) Instantaneous velocity vector of flow and instantaneous flame position in the combustion chamber.
(b) Instantaneous temperature distribution on a central section.
(c) Average temperature distribution on a central section.

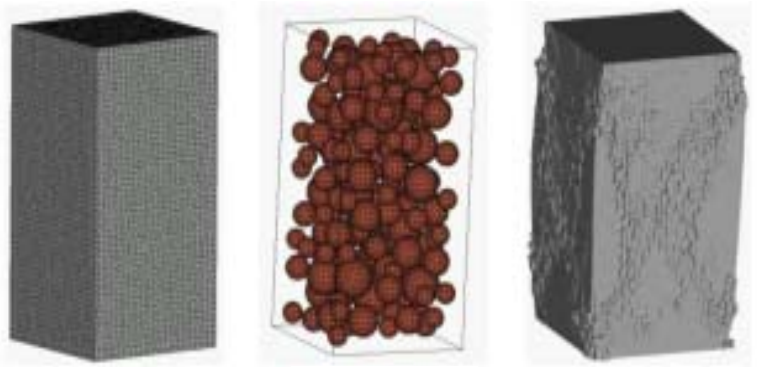
Application of *SMS-AMG*

Concrete Strength Study

SMS-AMG used in study of concrete breakage strength

1. Hokkaido University Structural and Geotechnical Engineering Division
2. Field of application: Concrete breakage strength study
   crack opening process simulation by meso scale model
3. Solution method: Rigid body spring model（RBSM）
4. Size of calculation: Several thousand times
of calculations in 250,000 to 400,000 unknowns
5. Calculation speed:
SMS-AMG on a Windows PC was approx. 10 times faster than an ICCG library on a super-computer in completing same calculations.
6. More info on http://www.hucc.hokudai.ac.jp/~m16120/hybridlab/index.htm

SMS-AMG enabled
10 times faster calculation by a PC than a supercomputer



Model Surface | Aggregate inside the model | Analysis Result

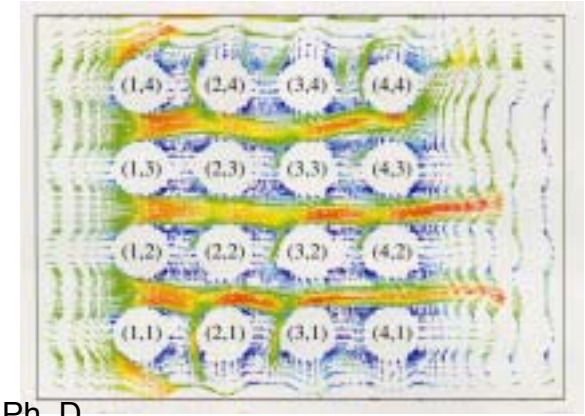
3D RBSM Analysis
75 x 75x 150mm Concrete
Compression Analysis Result
(48,258 elements) loading plate is fixed horizontally

Source: Nagai, et al. NUMERICAL SIMULATION OF FRACTURE PROCESS OF CONCRETE MODEL BY RIGID BODY SPRING METHOD.

Application of *SMS-AMG*

Nuclear Power Plant Fuel Rods
Fluid Force Vibration Analysis

Saitama Institute of Technology

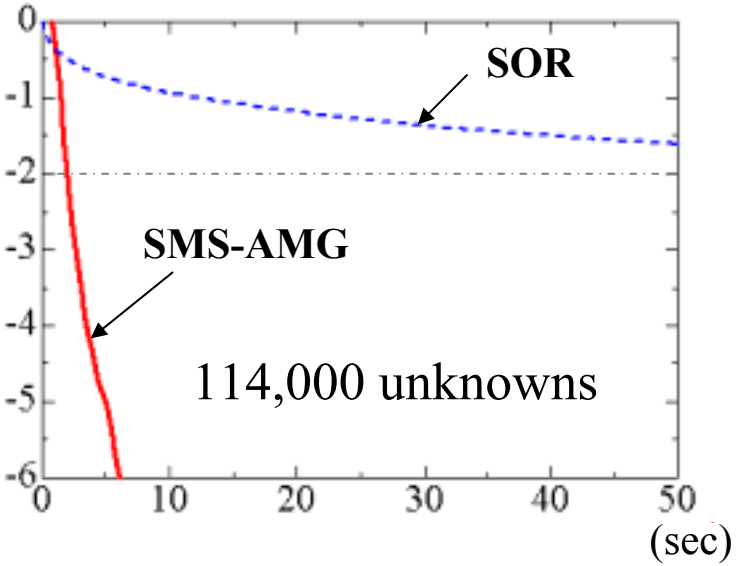


✓ **Calculations to achieve same accuracy（10^{-2}）were speeded up 50 to 100 times.**

Comment from Hisashi Hishida, Ph. D,
Saitama Institute of Technology

[Field of application]: CFD
Nuclear power plant fuel rods fluid force vibration analysis

[Calculation issues]
Trial of some large models took a few months to be solved by conventional iterative methods, but their calculation was significantly speeded-up by SMS-AMG.



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Application of *SMS-AMG*

Plamedia Corporations

✓Previously insoluble problems solved in high speed

✓3.5 times faster calculations

Study of heat and flow inside a spiral mandrel die and quantitative analysis of residence time profile by using 3D finite element analysis

Table Comparison of CPU time

Model	Eqn. 52357 Non0s. 4525296	Eqn. 143386 Non0s. 12144782
Direct solver	283.4 sec	Error (Memory over)
Iterative solver SMS AMG	81.9 sec	100.8 sec

(Monitor: Pentium IV PC, 2.53GHz, 2Gb)

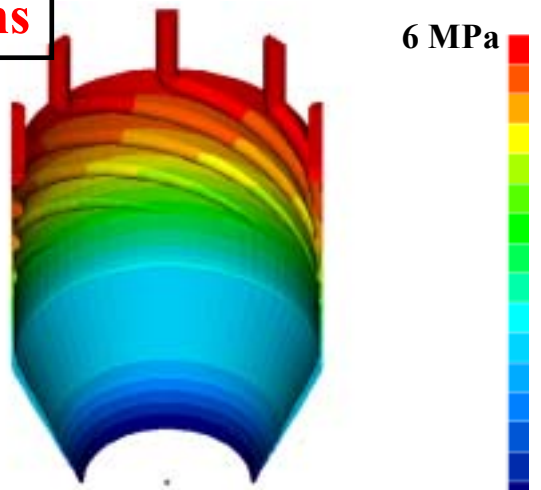
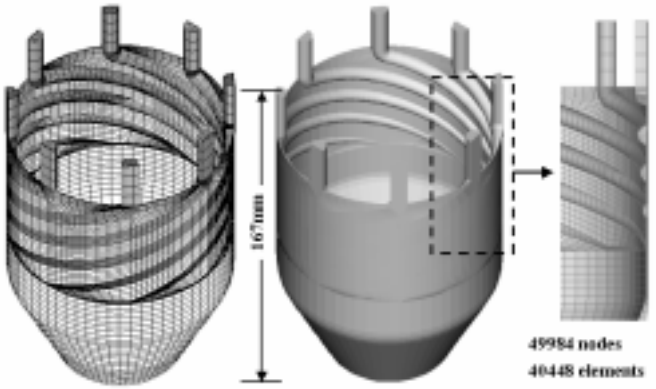


Fig. Pressure distribution



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Application of *SMS-AMG*

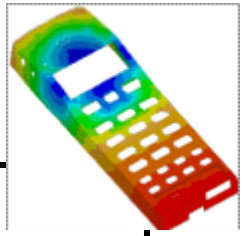
PLAMEDIA 株式会社 プラメディア

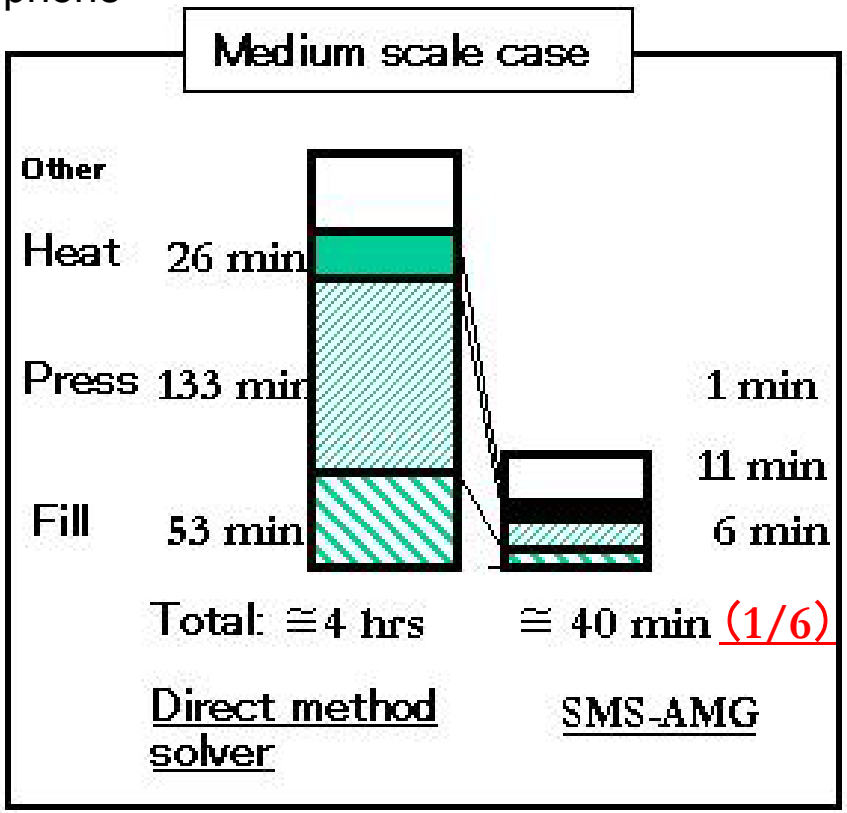
Plamedia Corporation

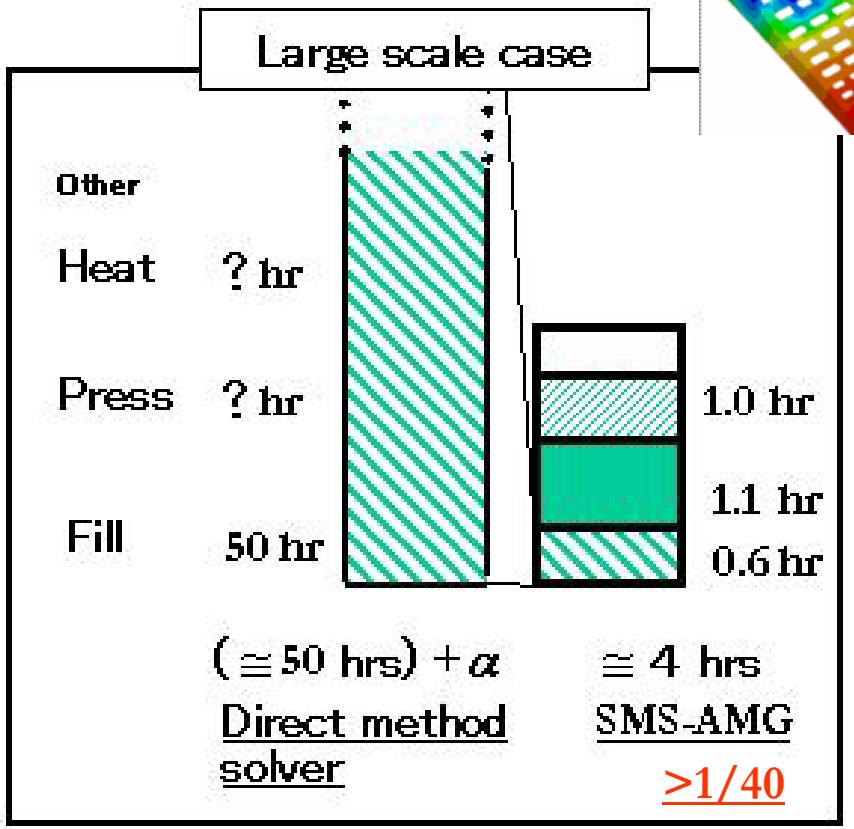
[Field of application]
Mold flow analysis of handheld phone

✓**Previously insoluble problems solved in high speed**

✓**6 times faster than famous Direct method**







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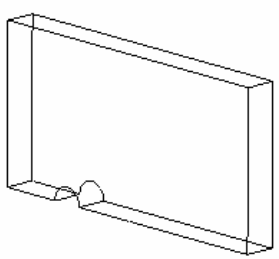
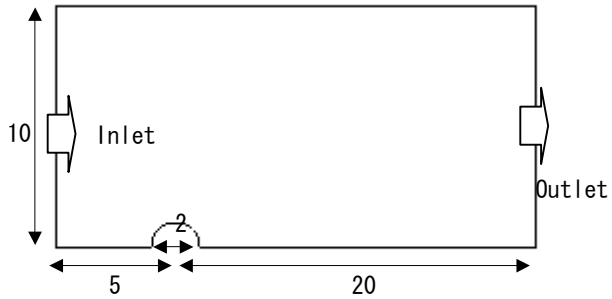
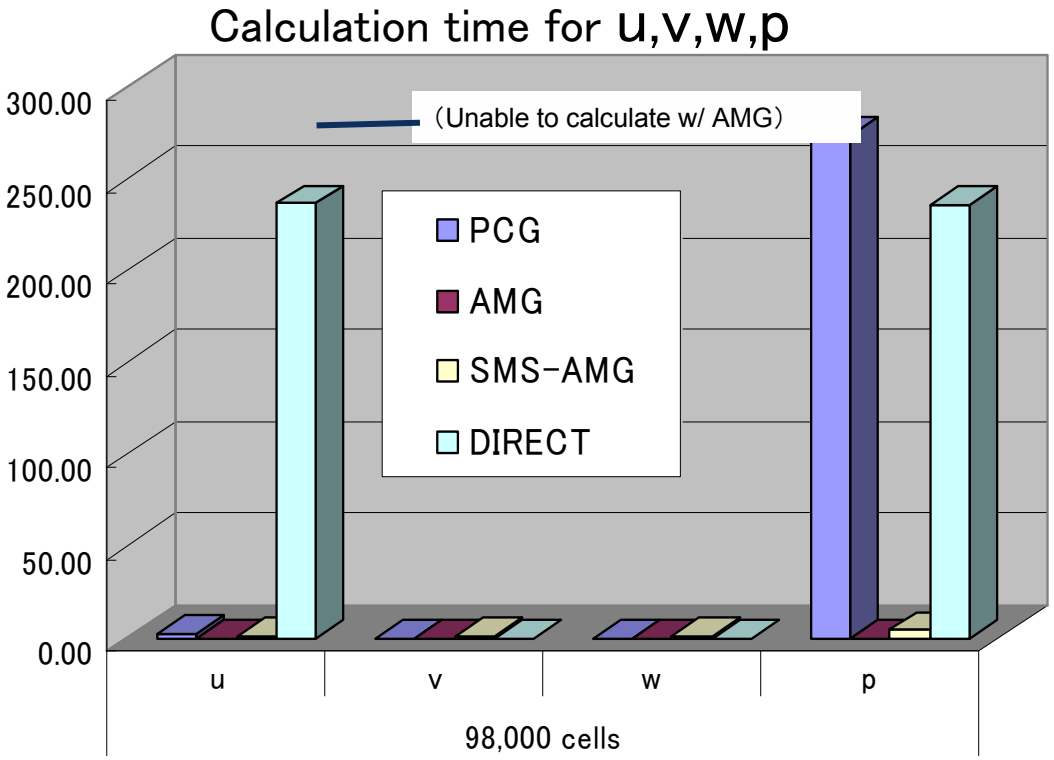
Application of *SMS-AMG*

CRC Solutions Corporation

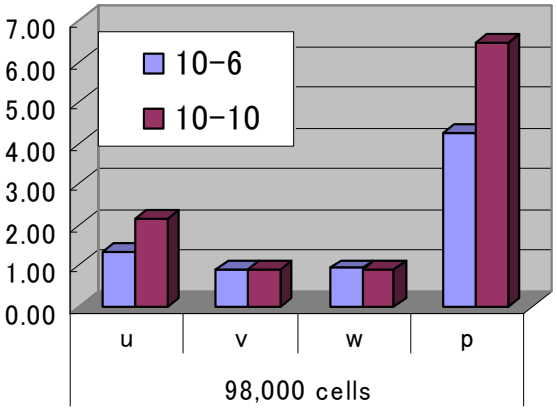
SMS-AMG is integrated into FINAS/CFD code







Inflow: Constant x-velocity(18.2)
Outflow: Gauge pressure = 0
Kinematic viscosity: 1.5×10^{-5}
Density: 1.0
Boundaries: Cylindrical surface has non-slip, other surfaces have slip condition.



Calc. Time for different convergence level

Application of *SMS-AMG*

The Institute of Space and Astronautical Science (ISAS) of Japan

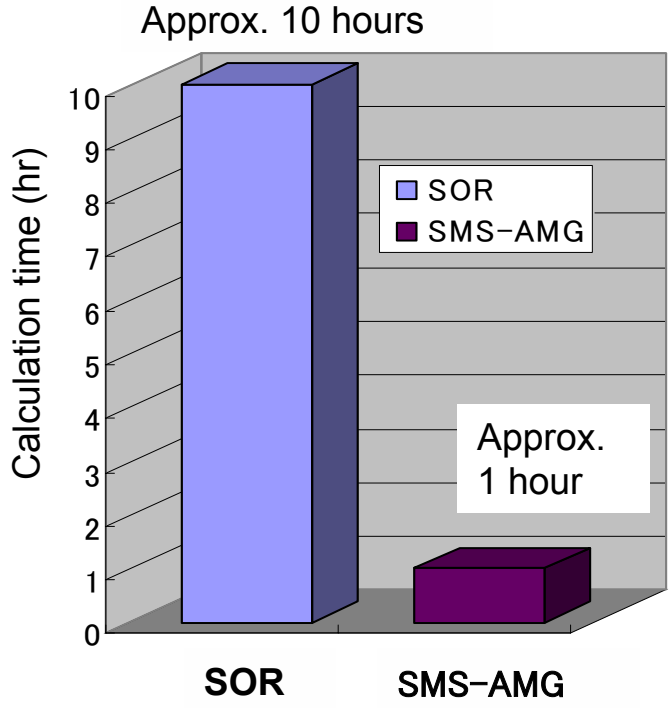
No. of elements: Approx. 40,000
Reynolds number: 1,000
Unknowns: 40,000
Conv. Criterion: 1E-4
Method: MAC method

Better convergence performance was observed at the beginning of calculation where instability is often observed. One of SMS-AMG's advantages is that it can carry out calculation more stably than SOR even with a larger time step.

Comment from user

✓**Previously insoluble problems solved in high speed**

10 times faster than before



Application of *SMS-AMG*

MU TECH Co., Ltd.

IEEJ's 3D static electromagnetic model for verification purpose

Iron core and coil model, one-eighth symmetric part

Hardware specifications

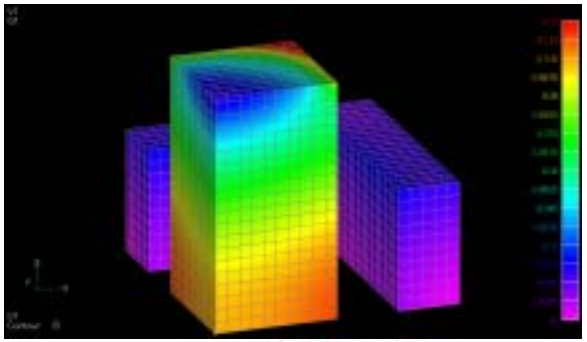
HP workstation x1100 / Pentium4
2GHz/ RAM 786MB, Hard disk 32GB

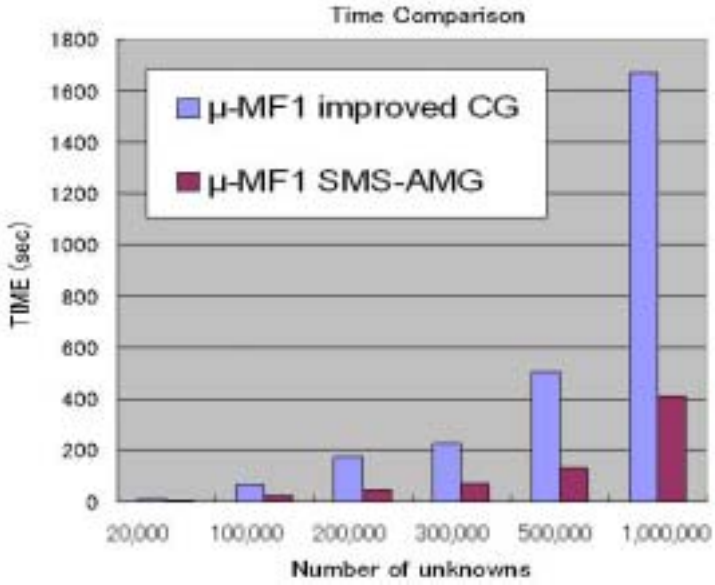
PC version of SMS-AMG from VINAS is capable of delivering its high performance in electromagnetic analyses through our performance tests.

Speed of calculation is far above what can be expected from conventional solution methods. With appropriate computer hardware, SMS-AMG is expected to be able to handle calculations with DOF exceeding 2 million.

User Statement

✓Four times faster in calculation of 1 Million DOF problem





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Visual Integration & Numerical Analysis Systems

Application of *SMS-AMG*

Electromagnetic Analysis for Superconductors

Yokohama National University raduate School of Engineering Laboratory of Dr. Amamiya

Field of application:
Electromagnetic field analysis for superconductors (Eddy current analysis of conductors with non-linear conductivity

Governing equation: Maxell equation

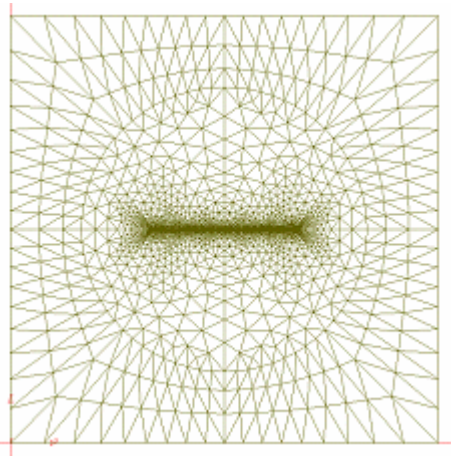
Analysis method: T-Ω method

Analysis characteristic: Aspect ratio of 2000 for the grid calculation of highly nonlinear problems for yttrium family superconductor materials

URL for laboratory: http://www.rain.dnj.ynu.ac.jp/

✓**Much faster calculation speed compared to conventional matrix solution method**

✓**Capable of calculating insoluble grids/analysis conditions**



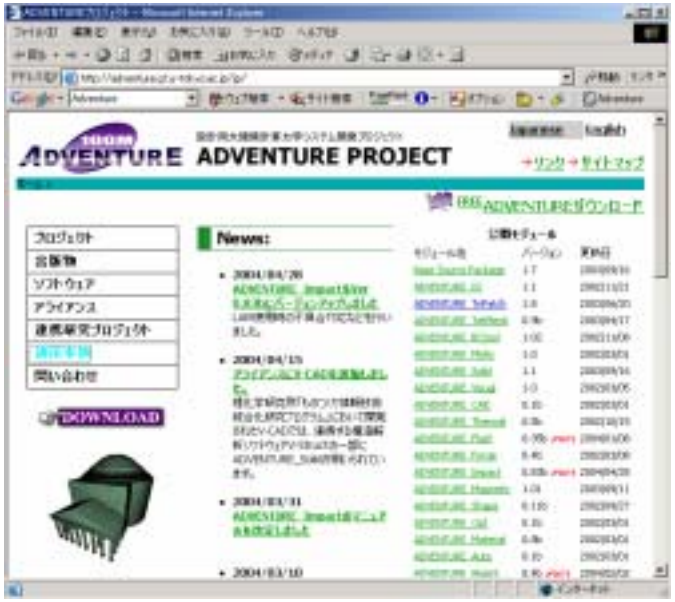
Calculation Time Comparison-Overall Analysis Calculation Time

	Approx. 26K Unknowns	Approx. 400K Unknowns
SMS-AMG	about 7,000 sec. convergence: 1.0E-10	about 130,000 sec. (about 36 hrs.)
BiCG	About 10,000 sec. convergence: 1.0E-6	Unable to calculate

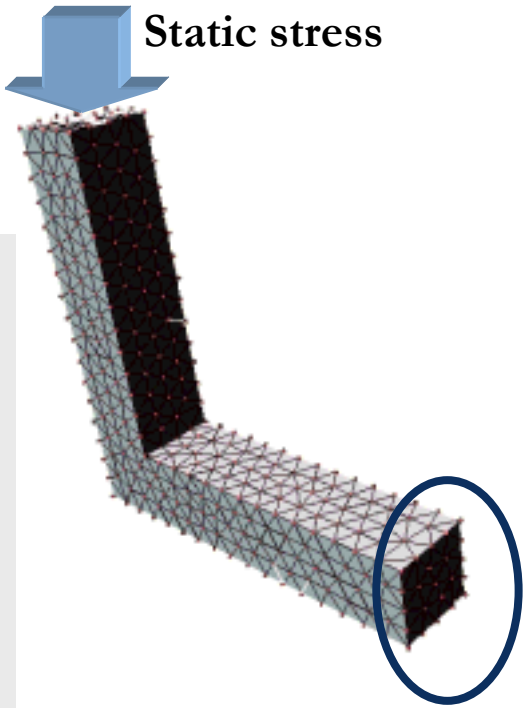
Comment: I think that introduction of SMS-AMG paved the way to analysis of the next-generation super conductor materials.

Application of *SMS-AMG*

Applied to ADVENTURE PROJECT



On trial applying SMS-AMG Version 2 to "Adventure Solid," improvement by applying SMS-AMG was verified.
Analysis field: Static elastic analysis



1997 - March 2002 as one of five projects in the "Computational Science & Engineering" field selected by the "Research for the Future (RFTF)" program sponsored by the Japan Society for the Promotion of Science (JSPS).

An advanced general-purpose computational mechanics system for large-scale analysis and design

Since 1st April 2002, we have reformed the ADVENTURE project into the Open Source Software Development Project and have continuously maintained and improved the system while applying it to practical engineering problems.
(Quotation from ADVENTURE web-site)

Application of *SMS-AMG*

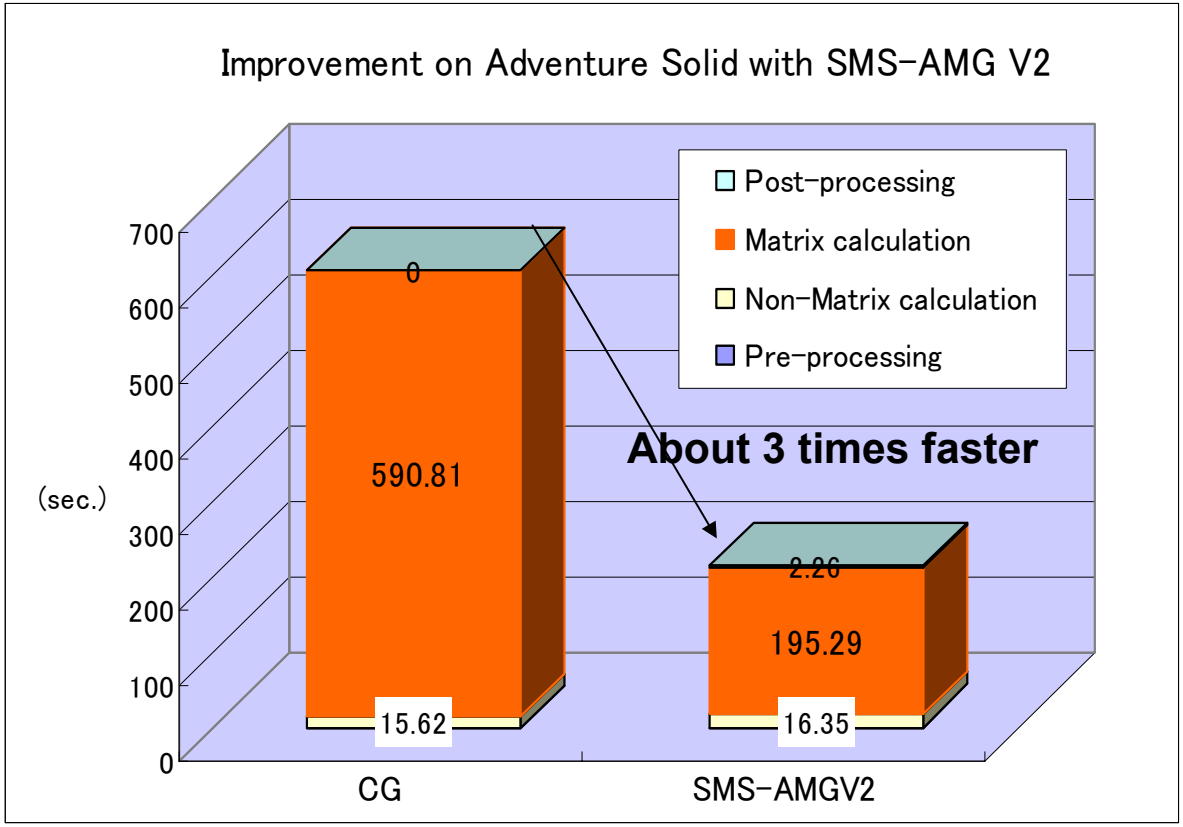
Static Stress Analysis

Applied to ADVENTURE PROJECT

Number of Nodes	76392
Number of Elements	38270
Number of DOFs	229176
Number of DOFs (Not restricted)	227580
Number of Diagonal Elements (ND)	227580
Number of Non-diagonal Elements (NS)	9658308

About 40,000 elements
About 230,000 unknowns

	CG	SMS-AMGV2
Pre-processing	0	0.9
Non-Matrix calculation	15.62	16.35
Matrix calculation	590.81	195.29
Post-processing	0	2.26



Application of *SMS-AMG*

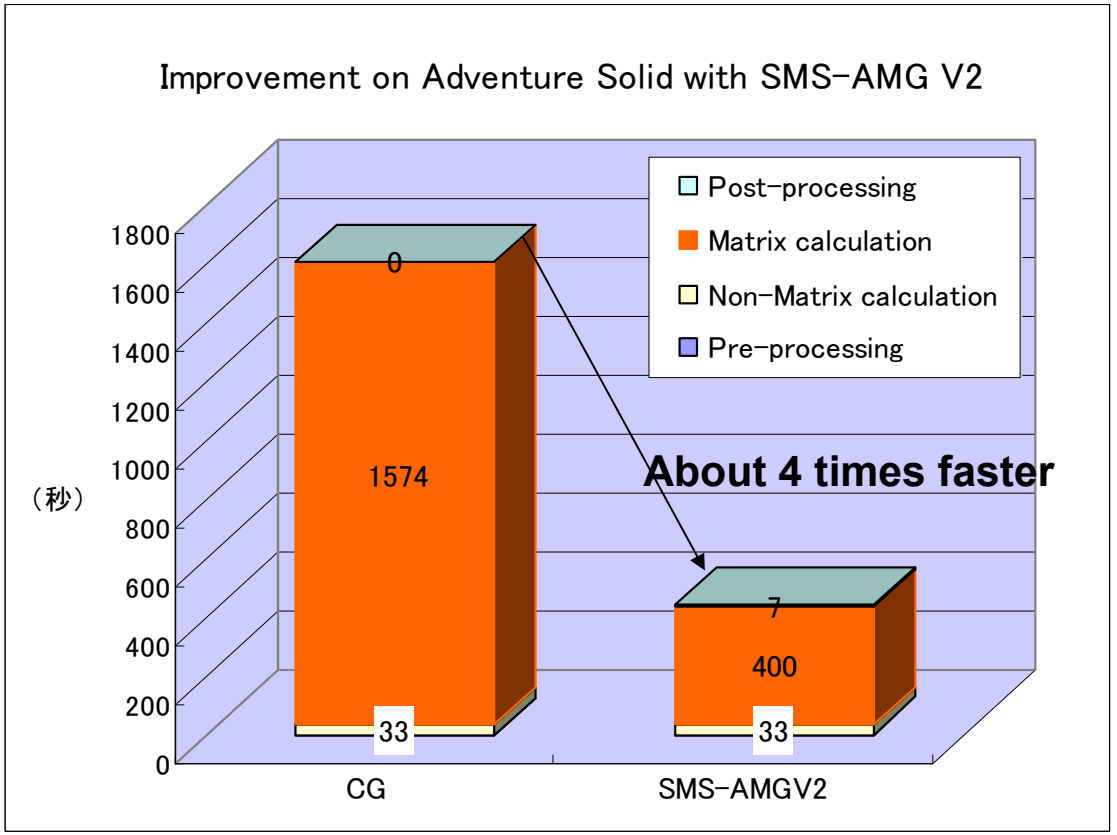
Static Stress Analysis

Applied to ADVENTURE PROJECT

Number of Nodes	150283
Number of Elements	60212
Number of DOFs	450867
Number of DOFs (Not restricted)	448371
Number of Diagonal Elements (ND)	448371
Number of Non-diagonal Elements (NS)	19305126

About 60,000 elements
About 450,000 unknowns

	CG	SMS-AMGV2
Pre-processing	0	1.9
Non-Matrix calculation	33	33
Matrix calculation	1574	400
Post-processing	0	7



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Visual Integration & Numerical Analysis Systems

For further information on *Super Matrix* Solver such as

- Benchmark Testing (BMT)
- Evaluation module
- Other inquiries

Please contact:

VINAS Co., Ltd.
Project Development Dept.

URL : **http://www.vinas.com** E-mail : sms@vinas.com